

Answer of Home work Problems

1) $x = cy e^{\frac{1}{xy}}$ 2) $xy - \frac{1}{xy} - 2 \log y = c$

3) $\log(x/cy^2) = 1/x^2 y^2$ 4) $xy - \frac{1}{xy} + \log \frac{x}{y} = c$

5) $\frac{1}{2} x^2 y^2 - \frac{1}{2} xy + \log \frac{x}{y} = c$ 6) $\log \frac{x}{y} - xy = c$

7) $\log(x^2/y) - 1/xy = c$ a) $\frac{1}{x^2}, \frac{1}{y^2}, \frac{1}{xy}$

2) $n = 2, x^3 + 2x e^y + y^3 = c$

3) $r^2/2 + r(\sin \theta - \cos \theta) = c$

Rule 6 If above 5 Method fails, Let $x^\alpha y^\beta$ be the integrating factor of $Mdx + Ndy = 0$, using the condition of exactness α, β can be easily found, provided the given equation is of the form

$$x^{a_1} y^{b_1} (m_1 y dx + n_1 x dy) + x^{a_2} y^{b_2} (m_2 y dx + n_2 x dy) = 0$$

where $a_1, b_1, a_2, b_2, m_1, m_2, n_1, n_2, h, k$ are all constants.

Ex: Solve $(y^2 + 2x^2 y) dx + (2x^3 - xy) dy = 0$

Ans: The given equation can be written as,

$$y(y dx - x dy) + 2x^2(y dx + x dy) = 0 \quad \dots (1)$$

Let the integrating factor of (1) be $x^h y^k$.

Multiplying (1) by $x^h y^k$ we get

$$(x^h y^{k+2} + 2x^{h+2} y^{k+1}) dx + (2x^{h+3} y^{k+1} - x^{h+1} y^{k+1}) dy = 0 \quad \dots (2)$$

which is Exact.

$$\therefore \frac{\partial}{\partial y} (x^h y^{k+2} + 2x^{h+2} y^{k+1}) = \frac{\partial}{\partial x} (2x^{h+3} y^k - x^{h+1} y^{k+1})$$

$$\begin{aligned} \therefore x^h (k+2) y^{k+1} + 2(k+1) x^{h+2} y^k \\ = 2(h+3) x^{h+2} y^k - (h+1) x^h y^{k+1} \end{aligned}$$

Equating the coefficients of like terms we get

$$k+2 = -(h+1), \quad 2(k+1) = 2(h+3)$$

$$\begin{aligned} \therefore h+k &= -3 & \Rightarrow h &= -\frac{5}{2} & k &= -\frac{1}{2} \\ h-k &= -2 \end{aligned}$$

$$\therefore \text{I.F.} = x^{-5/2} y^{-1/2}$$

Multiplying both sides by $x^{-5/2} y^{-1/2}$ to the given eqn. we get

$$\begin{aligned} x^{-5/2} y^{3/2} dx - x^{-3/2} y^{1/2} dy + 2x^{-1/2} y^{1/2} dx \\ + 2x^{1/2} y^{-3/2} dy = 0 \end{aligned}$$

$$\Rightarrow 4 d(x^{1/2} y^{1/2}) - \frac{2}{3} d(x^{-3/2} y^{3/2}) = 0$$

Integrating we get

$$6 x^{1/2} y^{1/2} - x^{-3/2} y^{3/2} = c$$

Home work

Solve

$$1) (2y dx + 3x dy) + 2xy(3y dx + 4x dy) = 0$$

$$2) (3x + 2y^2) y dx + 2x(2x + 3y^2) dy = 0$$